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Letter to the Editor

Further research on the Projective Covering Method. Reply to comments by A. Carpinteri, B. Chiaia and S. Invernizzi on the paper "Direct fractal measurement of fracture surfaces", *Int. J. Solids and Structures* 36, 3073–3084, by H. Xie and J. Wang

The authors are grateful to Professors A. Carpinteri, B. Chiaia and S. Invernizzi for their comments. The authors do not agree, however, with all points of the comment and offer the following reasons for each discussion point in order.

Extensive research shows that rock joint surfaces exhibit both self-similar and self-affine fractal behaviors, or rather the self-affine fractal behavior. Many researchers have tried to apply fractal concepts to the study of fracture surfaces, beginning with Mandelbrot's research (1982). In his pioneer work (Mandelbrot et al., 1984), two methods were used to determine a fractal surface dimension. The first one is a so-called profile analysis, in which an elevation profile across the fracture surface is analyzed, and the second is the silt island analysis, in which a flat polished section parallel to the nominal surface creates islands and lakes. Undoubtedly, these methods belong to the extension from one-dimensional measurement to two-dimensional measurement. However, such extension from one-dimensional measurement to two-dimensional situation is not sufficient to characterize the morphology of rock joint surfaces because of the extreme complexity and anisotropy of rock joint surfaces (Xie et al., 1997a, 1998a). Therefore the direct fractal measurements of fracture surface should be proposed.

It is well known that the covering method is a common way to perform the fractal measurement of a profile or surface. For a single profile across a rock joint surface, it is easy to estimate its fractal dimension using the direct covering method. It is very difficult, even impossible, however, to make a direct covering measurement for a two-dimensional rough surface. In this case, the Projective Covering Method (PCM) was proposed in the paper (Xie and Wang, 1999). The PCM to evaluate the fractal dimension of a fracture surface belongs to the class of the general divider methods. The authors believe that the term "covering" mentioned in this paper (Xie and Wang, 1999) is not misleading at all. Within the framework of fractal geometry, no matter which method is used to estimate the fractal dimension of a rough surface, the fractal dimension should be a real geometric fractal dimension. In addition, the method to estimate the fractal dimension should be simple and acceptable.

It is seldom possible to make an exact calculation of the actual area of the rough surface within the grid unit shown in Fig. 1 (Clarke, 1986) because, usually, the four points considered seldom lie on a plane. One possible solution is to estimate the approximate value of rough surface area. For example, in the triangular prism surface area method (Clarke, 1986; Muralha, 1995), the approximate area is estimated by calculating the area of four triangles. The elevation at the center of each grid cell is determined by linear interpolation (average) of the four heights of the adjacent points. In this case, the point at the center of grid cell determined by the triangular prism surface area method (Clarke, 1986) is not usually on the rough surface, and may even be far from the actual rough surface. It may lead to a large error. Based on this reason, a new covering concept was proposed in the PCM. Box covering

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method was considered in the projective plane of the fracture surface, its covering areas were estimated within mapping space between the projective plane and the real fracture surface. Although Eq. (1) in the PCM (Xie and Wang, 1999) is an approximate formula, every point for calculation of the approximate area can be assured to be on the rough surface. When the scale size $\delta \rightarrow 0$, the calculated fractal dimensions D_s will tend to become the real fractal dimension of rough surface. Therefore, it should be concluded that the PCM is quite different from the triangular prism method, or at least is a better modification of the triangular prism surface area method.

The authors agree that the scale mentioned in the paper was not small enough. It makes the fractal measurement lose some features of rock joint surface morphology, especially the local features. The authors believe it is one of the reasons why the measured fractal dimensions are always low. At present, the authors have improved this situation. The minimum scanning resolution can be as accurate as 12.5 μm.

The estimation of the fractal dimension of a natural rock joint surface was not conducted by self-affine measurement method in the paper. It is well known that, for an individual profile, the semi-variogram function can be used to determine the self-affine fractal dimensions (Kwasniewski and Wang, 1993, 1997; Xie, 1993; Xie and Pariseau, 1994; Xie et al., 1996, 1997a,b, 1998b, 1999). But for a two-dimensional surface, it is impossible to make a self-affine measurement because there is no such theoretical function. In this case, the authors had to use self-similar method (Xie and Sun, 1997; Xie et al., 1998a; Xie and Wang, 1998, 1999; Xie and Zhou, 1998). Although the self-similar dimension is quite low (lower than 2.2, Muralha, 1995; Xie and Wang, 1999) and not sensitive to the roughness, at least it can be concluded that this measurement method is the state-of-the-art in application of fractal geometry to the rock mechanics. The authors believe that if the self-affine function of a surface could be deduced from theory, this problem would be solved successfully.

In this paper (Xie and Wang, 1999), Eq. (9), showing the relation of fractal dimension between the Cartesian product of fractal sets and the individual fractal sets, was employed to express the relation between the fractal dimension of a surface and the sum of fractal dimensions estimated along two individual perpendicular directions. In the last decade, some researchers even Mandelbrot (1982) himself suggested that fractal dimension of a topographic surface can be obtained by adding 1.0 to the fractal dimension obtained for a single profile from that surface. In the present research, the calculated results shown in Table 1 (Xie and Wang, 1999) indicated that the fractal dimension of a rough surface fall between value obtained by adding 1.0 to the fractal dimension of individual profile and the sum of fractal dimensions estimated along two individual perpendicular directions. From the viewpoint of theory, the conclusion should be correct.

It should be pointed out that the Cartesian product of fractal sets is not the actual fractal surface at all. However, from the viewpoint of theory, we should consider how to construct a fractal surface using two fractal sets of profile. Usually, three ways, i.e., special Cartesian product, and fractional Brownian motion surface and star product fractal surface proposed by us in recent days can be used to approach the natural fractal surface.

Generally speaking, a fractal surface is a special set in three-dimension Euclidean space R^3. In general, the natural fracture surface can be regarded as a binary function, $z=f(x, y)$. Suppose A is a fractal profile in plane ZOX, $A=\{(x, y):x\in[c, d], z=g(x)\}$, while B is an interval in R, $B=[a, b]$, then a simple fractal surface F can be described by Cartesian product

$$F = A \times B = \left\{(x, y, z):x \in [c, d],\ y \in [a, b],\ z = g(x)\right\}$$

It means the fractal surface F can be obtained by movement of fractal profile A along the straight line B. In this case, the fractal dimension of a simple Cartesian product can be given by (Falconer, 1990)

$$\dim(A \times B) = 1 + \dim A$$

Fractional Brownian motion surface could be applied to simulate the morphology of the fracture surface. For any real number in the range of $0 < H < 1$, if (1) with the probability 1, the two-dimensional Brownian function $B_H(x, y)$ is a continuous function for any (x, y); (2) for a given (x, y), $(h, k) \in R^2$, the probability distribution function of increment $B_H(x+h, y+k) - B_H(x, y)$ is a Gaussian or normal distribution with zero average increment and a variance of increment of $(h^2+k^2)^H$, i.e.,

$$P(B_H(x+h, y+k) - B_{\mathrm{H}}(x, y) \leqslant z) = (2\pi)^{-1/2}\left(h^2+k^2\right)^{-H/2}\int_{-\infty}^{z} \exp\left(\frac{-r^2}{2(h^2+k^2)^H}\right) \mathrm{d}r$$

then $\{(x, y, z):(x, y) \in R^2, z = B_H(x, y)\}$ is called the Brownian surface with the exponent of H. It can be strictly proved that the Hausdorff dimension and box dimension of the Brownian surface with the exponent of H are equal to $3-H$ (Falconer, 1990).

Some researchers assumed the fractal dimension of naturally developed rough surface to be the fractal dimension $3-H$ of Brownian motion surface. Nevertheless, the Brownian motion is a completely random process, while the rock joint surface is not completely stochastic. Therefore the simulation of natural surfaces using Brownian motion may cause an obvious error. For this case, the fractal dimension obtained by $3-H$ usually exceeds the actual fractal dimension.

A third way to theoretically construct the fractal surface is the so-called the star product of two fractal profiles proposed by Xie and Feng (1999). Suppose A and B are the fractal profiles in plane ZOX and plane YOZ, respectively, i.e. $A = \{(x, z):x \in [a, b], z = g(x)\}$, $B = \{(y, z):y \in [a, b], z = h(y)\}$. The star product F of A and B is defined as movement of A along B or movement of B along A, i.e.

$$F = A * B = B * A = \left\{(x, y, z):(x, y) \in [a, b] \times [c, d],\ z = g(x) + h(y) - g(a)\right\}$$

It has been proved that (see Xie and Feng, 1999),

$$\dim F = 1 + \max(\dim A,\ \dim B)$$

The authors suggest that a description of the natural fractal surface can be approached by the special Cartesian product and Brownian motion surface, or rather star product of two fractal individual profiles. it is indicated that the fractal dimension of star product of two fractal sets is better approximate to the fractal dimension of rock joint surface than Cartesian product. Of course, the star product fractal surface is more regular than the natural fracture surface, so its dimension should be less than the dimension of the fracture surface. In other words, the dimension of the fracture surface (such as the fracture surface of rock) should be larger than the dimension of every sectional profile plus one.

The statement by A. Carpinteri, B. Chiaia and S. Invernizzi that the value of fractal dimension of a wide class of fracture surfaces $D = 2.2$ is universal is not true. In Bouchaud et al.'s paper (1990), it was concluded that, within the experimental error, the fractal dimensions of the four studied samples do not depend either on the rupture mode or on the fracture toughness value, but are centered on an average value 2.2. It should be noted that, the estimation method of fractal dimension in the paper (Bouchaud et al., 1990) is not accurate but approximate.

In fact, the morphologies (i.e. roughness) of rock joint surfaces differ widely from one another. The fractal dimensions, as an indicator of roughness of rock joint surfaces, should be very different too. The present problem is how to make the fractal dimension be sensitive to the roughness. Objectively speaking, the direct measurements such as the PCM should be the most reliable method. The results in this paper (Xie and Wang, 1999) may not be very accurate because of the limitations of measurement resolution, but they are the most believable.

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